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TURBOMACHINERY DETERIORATION MODELING BASED ON IN-SERVICE DATA

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Abstract

In this paper, performance deterioration models for selected gas turbine components are derived from a statistically relevant set of in-service data. Therefore, deterioration trends are computed using model-based performance analysis. Deterioration is expressed using modifiers on component maps to match the measurements. Averaging the deterioration trends of comparable data sets, baselines for power plant groups are established. Utilizing non-linear regression analysis, two different models are fitted to these baseline deterioration trends. The predictive capability of the employed regression method is evaluated by extrapolation. The results show, that the power function is a suitable statistical deterioration model for the evaluated data in case of power plant group baselines as well as individual gas turbines. Moreover, this function may be used for prediction of performance deterioration, too. However, a certain amount of snapshots is needed to get stable extrapolation results.

Nomenclature**Symbols**

a, b, c	model parameters
m	health parameter
\hat{m}	predicted health parameter
\bar{m}	mean of n health parameters
\hat{n}	predicted total number of cycles
i, n, k, y	running indices
R^2	coefficient of determination
t	cycle number
ε	residual
ξ	prediction error
σ	standard deviation
\hat{t}	predicted remaining number of cycles

Subscripts

cap	capacity
eff	efficiency
in	initial

ref reference

Acronyms

CFD	computational fluid dynamics
CI	confidence interval
DS	data sets
HPC	high pressure compressor
HPT	high pressure turbine
LPC	low pressure compressor
M	mean
Mdn	median
RMSE	root-mean-square error
SD	standard deviation
SV	shop visit

Introduction

During service, the performance of gas turbines degrades due to deterioration mechanisms such as fouling, erosion, corrosion and abrasion [1]. Maintenance activities may result in performance and life recovery. In this context, the transition from time-based maintenance to condition-based maintenance may increase the profitability of gas turbines [2]. Therefore, engine condition monitoring systems that can diagnose and predict performance conditions with adequate precision are required [3, 4]. Such systems need to include deterioration models, which describe the relationship between the operating parameters of a gas turbine and its performance loss over time. In this paper, such models are derived from in-service data.

Deterioration Modeling

Two different approaches can be chosen for modeling of performance deterioration. Following a deterministic or bottom-up approach, the underlying physical deterioration mechanisms like fouling, erosion and corrosion are modeled. Based on a set of operating parameters and gas path quantities these models quantify the damage done to the component, which can be expressed by a change of the gas path geometry. The resulting change of the component performance may then be determined using computational fluid dynamics (CFD). However, the complexity of

the deterioration mechanisms only allows the derivation of generalized phenomenological models, which neglect geometric effects and the interaction of the deterioration mechanisms. Moreover, semi-empirical influence factors are commonly used for conversion of the damage information provided by these models into a change of the component performance. Thus, the computationally expensive use of CFD can be avoided. Assuming future operating parameters e.g. load requirements and ambient conditions, these models may be applied to predict performance deterioration [5].

Following a statistical or top-down approach, the deterioration model is derived from in-service data. Using performance synthesis and methods of time series analysis, deterioration trends are computed. Employing regression techniques, mathematical functions are identified which are suitable to describe these trends [6, 7]. Hence, in this case the deterioration model, which corresponds to the regression model, is purely mathematical in nature. By extrapolation of these models, the performance deterioration may be predicted. Following a top-down approach, the operating parameters are implicitly included in the deterioration model. Hence, in this case the prediction is based on the assumption that the operating parameters will not change in the future.

Analysis Methodology

Selection of In-Service Data

In-service data of two different types of two-shaft gas turbines are investigated. In order to ensure the relevance of the findings, the analyzed data sets have to be comparable. Therefore, two groups of power plants named “GT-A” and “GT-B” are defined according to the following criteria:

- In a group all gas turbines have a comparable build standard
- In a group the operating conditions of all gas turbines are comparable
- In a group all gas turbines have a comparable standard of instrumentation and data acquisition
- In a group all gas turbines have a similar maintenance history

The importance of the last aspect is indicated in Fig. 1, in which the similarity of the maintenance history is defined by the number of major maintenance activities requiring a shop visit. The deterioration gradient of the high pressure compressor (HPC) decreases once a gas turbine has been overhauled in the shop. According to the last cycle’s 95 % confi-

dence interval of the mean, the difference in the deterioration characteristic is statistically significant.

It must be noted that there is no information available concerning the scope of maintenance activities. Hence, the comparability of data sets obtained from gas turbines which have already been overhauled is somewhat affected. Therefore, in case of the power plant group “GT-A” only gas turbines in new condition are considered in the further research. However, in case of the power plant group “GT-B”, a sufficient amount of data sets is available only for gas turbines which have already been to the shop once.

Selection of Health Parameters

The gas turbines of group “GT-A” and “GT-B” feature different sets of measurands. In both cases the number of measurands is lower than the number of health parameters. Hence, the full set of health parameters cannot be calculated by means of performance synthesis and conventional analysis methods. Therefore, the number of health parameters is reduced in order to obtain a closed system of equations in the analysis process. This introduces some uncertainty, since the parameter reduction implies the assumption that the neglected parameters are not subject to deterioration [8]. Only the results of the health parameters not being affected by this approach are reported. They are shown in Tab. 1.

Tab. 1: Investigated health parameters

Power Plants Group	Health Parameters
GT-A	Efficiency of Low and High Pressure Compressor
GT-B	Capacity of High Pressure Turbine

Calculation of Health Parameters

The performance degradation during service has to be assessed for each gas turbine individually. This is due to the fact, that each gas turbine is exposed to its individual load collective. This includes ambient conditions as well as power settings and number of load changes. For computation of the performance deterioration, the measured gas path data is compared to respective expected data calculated by a performance model of the average, new, production standard gas turbine. Since the in-service data is acquired during steady-state operation, transient effects [9] do not need to be considered. Modifiers are applied to the maps of component efficiency and component capacity in order to match the measurements. These modifiers, which represent the health parameters m , are generally assumed to be independent of the oper-

ating point [10]. This is important, since changes in component performance lead to a rematch of the component operating points [1].

Each gas turbine is assembled by an individual set of hardware. This represents a random occurrence within a population of production standard gas turbines. Hence, there is variation in the baseline performance among the gas turbines of a power plant group. To establish a common performance baseline, acceptance test measurements are used to compute initial health parameters m_{in} for each gas turbine. The effective performance deterioration Δm at a given cycle t is defined as

$$(1) \quad \Delta m_t = m_t - m_{in}, \quad t = 1 \dots n.$$

The evolution of the parameter Δm in time is referred to as “deterioration trend”.

Method of Data Reduction

The analysis of the in-service data results in plots of the health parameters versus cycle number. Such a plot is shown in Fig. 2 in case of the individual gas turbine “GT-A-01”. The significant scatter is due to measurement and modeling uncertainties. The latter includes the assumption that the fuel heating value (FHV) is constant. In practice, this quantity varies [16]. Health parameters located outside of the two-sigma confidence interval around the moving average are considered outliers. In order to improve the results of the subsequent least-squares regression analysis, these outliers are filtered out. Furthermore, the moving average is applied to the deterioration trends to further reduce the scatter. The remaining scatter is tolerated to improve time resolution.

Although the in-service data has been selected carefully, there is considerable variation in the severity of deterioration within the power plant groups. Establishing a baseline deterioration trend of the respective power plant type, data from multiple gas turbines is averaged. In so doing, the impact of random or quasi-random errors is reduced and the sum of the systematic deterioration mechanisms is isolated. In Fig. 3 such a baseline deterioration trend is shown in case of the efficiency of the low pressure compressor (LPC). Furthermore, the 95 % confidence interval of the mean is shown.

Regression Analysis

Two different functions are fitted iteratively to the baseline deterioration trends using a non-linear least squares method. One is a power function of the form

$$(2) \quad \Delta \hat{m}(t) = a \cdot t^b + c,$$

where a , b and c refer to the model parameters ($a, c \in \mathbb{R}, b > 0$). In previous research [6, 11, 12], the power function has proven to be appropriate to reproduce the evolution of performance deterioration in time. Additionally, a function involving the hyperbolic tangent is investigated in case of the LPC efficiency

$$(3) \quad \Delta \hat{m}(t) = a \cdot \tanh(b \cdot t) + c,$$

where $a, c \in \mathbb{R}, b > 0$. The hyperbolic tangent is suitable to reproduce transitions to very low deterioration rates. Furthermore, as can be seen in Fig. 4, during the initial stage of deterioration this function can reproduce nearly linear changes whereas the power function behaves strongly non-linear. Due to the initial calibration according to Eq. (1) the deterioration trends start at zero, hence the model parameter c is not considered in this paper.

There are significant aleatoric uncertainties underlying the deterioration trends originating from measurement and modeling uncertainty. Moreover, some of the deterioration trends show epistemic uncertainties due to seasonal effects which manifest as oscillations. Hence, the minimization of the sum of squared residuals is a non-smooth and potentially multimodal optimization problem, which cannot be solved reliably by the exclusive use of gradient based optimization methods. Therefore, a hybrid approach is chosen: In order to identify a likely candidate for the global minimum, an evolutionary algorithm for global optimization is run first. Subsequently, a local optimization is performed to efficiently refine the result of the global optimization if possible. For local optimization, a generalized reduced gradient algorithm is employed [13,14].

The goodness of the fits is expressed by the coefficient of determination R^2

$$(4) \quad R^2 = 1 - \frac{\sum_{t=1}^n (\Delta m_t - \Delta \hat{m}(t))^2}{\sum_{t=1}^n (\Delta m_t - \Delta \bar{m})^2},$$

where $\Delta \bar{m}$ refers to the mean of the time series covering n observations. $\Delta \hat{m}$ represents the prediction of the model. The nominator in Eq. (4) corresponds to the residual sum of squares, the denominator refers to the total sum of squares which is proportional to the variance within the data. Hence, R^2 describes the amount of the variance which can be explained by the model. A R^2 value of 1 indicates that the model perfectly fits the data. Applied to non-linear regression problems, R^2 can yield values below 0 also. The

goodness of the fits is further expressed by the root-mean-square error (*RMSE*), which is also referred to as “standard error”

$$(5) \quad RMSE = \sqrt{\frac{\sum_{t=1}^n (\Delta m_t - \Delta \hat{m}(t))^2}{n}}$$

Apart from that, given an ideal regression model the residuals are normally distributed.

Prediction of Health Parameters

The predictive capability of the power function is investigated by extrapolation of data sets of individual gas turbines. Regression analyses of the deterioration trends are performed at equidistantly distributed cycles t_i . Thus the model parameters a_i and b_i as well as the predicted health parameter value at the last cycle $\Delta \hat{m}_i(t=n)$ are determined. After each fit, the relative prediction error is calculated

$$(6) \quad \xi_i = \frac{(\Delta \hat{m}_i(n) - \Delta \hat{m}_y(n))}{|\Delta \hat{m}_y(n)|}, \quad i = 1 \dots y$$

where $\Delta \hat{m}_y(n)$ refers to the predicted health parameter at the last cycle n of the final fit y . Furthermore, the number of cycles $\hat{\tau}_i$ until reaching the final performance degradation $\Delta \hat{m}_y(n)$ is estimated

$$(7) \quad \hat{\tau}_i = \hat{n}_i - t_i, \quad i = 1 \dots y$$

Therefore, the power function is rearranged to predict the total number of cycles until reaching $\Delta \hat{m}_y(n)$

$$(8) \quad \hat{n}_i = \left| \frac{\Delta \hat{m}_y(n)}{a_i} \right|^{1/b_i}, \quad a_i \neq 0, \quad i = 1 \dots y$$

As can be seen in Fig. 4, the gradient of the power function is flattening with progressing deterioration. Hence, even small fitting deficiencies may result in significant errors in prediction of the remaining number of cycles. An optimum fit is ensured using the described hybrid approach.

Results and Discussion

Uncertainty in Health Parameter Calculation

In order to compute a measure of the aleatoric uncertainty, the standard deviation of the residuals is computed for all health parameters and data sets. The residuals ε_i are defined as

$$(9) \quad \varepsilon_1(t, k) = \Delta m_t - \Delta \bar{m}(t, k), \quad t = k \dots n$$

where $\Delta \bar{m}(t, k)$ corresponds to the moving average at the cycle t covering a certain number of cycles k . According to Fig. 5, the uncertainty of the efficiency of the HPC is lower compared to the LPC. This result is in line with the findings presented in [15]. The uncertainty of the high pressure turbine (HPT) capacity is largest. An influence parameter study was conducted considering the parameters which are most important to the employed heat balance analysis method. In this study, the uncertainty levels given in [15,16] were used. According to Fig. 6, the uncertainty of the HPT capacity results mainly from the assumption that the FHV is constant. In practice, there is some variation in this value [16]. Apart from that, the uncertainty of the HPT capacity results from the uncertainty associated with the measurement of the exhaust gas temperature (EGT) and the fuel flow (FF), too. It has to be noted, that the comparability of the results shown in Fig. 5 is somewhat affected due to the different sizes of data set population.

Uncertainty in Baseline Deterioration

In order to investigate how the data quantity influences the consistency of the baseline deterioration trends, the data sets were categorized into three different groups covering different numbers of cycles. After filtering each data set using the moving average, the mean of the final deterioration value is calculated for each group. In Fig. 7, these values are shown. Furthermore, the 95 % confidence interval for the means is shown. Since there are no statistically significant differences between the mean values at the given cycle numbers, the baseline deterioration trends of the three groups can be considered consistent. The uncertainty of the mean is the largest in case of the HPT capacity. This is reflected by the confidence intervals of this parameter depicted in Fig. 7. One possible reason for this is that the type-GT-B gas turbines have undergone one maintenance shop visit already. Therefore, different scopes of maintenance activities and different hardware may have led to different deterioration characteristics. Apart from that, in case of the type-GT-B gas turbine a smaller number of data sets is available, which additionally increases the confidence interval of the mean. As can be further seen in Fig. 7, the difference of the efficiency deterioration between the LPC and the HPC is throughout of statistical significance.

Deterioration Models

In Fig. 8 histograms of the fitting residuals are shown. In case of the LPC efficiency, the residual distribution is in good agreement with the normal distribution for both of the deterioration models. This indicates a good calibration of the regression models.

However, the distributions are slightly shifted to the left side as indicated by the negative values of the mean. This is mostly due to the fact, that the regression models have been fitted to the filtered data rather than the raw data so as to reduce the negative impact of outliers. As the application of the moving average leads to a time delay, the model output at a certain cycle number $\Delta\hat{m}_t$ is systematically too small. Since in this case the residual is defined to be

$$(10) \quad \varepsilon_2(t, k) = \Delta m_t - \Delta\hat{m}(t - k), \quad t = k \dots n$$

the mean of the residuals is negative. According to Fig. 8, the power function is advantageous over the hyperbolic tangent, since it results in a residual distribution which has less skew, less offset and a slightly lower standard deviation. As for the HPC efficiency, the residual distribution deviates considerably from the normal distribution as indicated by the significant positive skew. Therefore, there are systematic effects underlying the HPC efficiency degradation which cannot be reproduced by the power function. For the same reason as in case of the LPC, there is a negative shift within the residual distribution. Although being skewed slightly positive, the residual distribution of the HPT efficiency is in reasonable agreement with the normal distribution. According to Eq. (10), the systematic underestimation of the respective deterioration model results in a positive shift of the residual distribution since the HPT capacity increases due to deterioration.

In Fig. 9 an overview of the different baseline deterioration trends together with the fitted power functions is shown. The R^2 values, which are throughout close to 1, indicate a good fit in all cases. According to the R^2 and the $RMSE$ value, the goodness of fit is best for the LPC efficiency. The reason for this is, that according to Fig. 8 the residuals of the LPC efficiency are normal distributed. Hence, most of its scatter can be efficiently filtered by application of the moving average. In contrast, there are considerable systematic and seasonally occurring effects in case of the HPC efficiency. In this case, the moving average filter is less effective. This translates into a worse goodness of fit of the respective baseline deterioration trend. As can be further seen in Fig. 9, the HPC efficiency is significantly stronger affected by deterioration than the LPC efficiency. This is in accordance with [17], where it was observed that compared to the LPC the HPC efficiency is more susceptible to airfoil contour erosion and to increasing blade tip clearances. The HPT capacity increases due to deterioration. This is a result of the increase of the HPT's effective throat area [1, 17]. Compared to the other two health parameters, the LPC efficiency shows a rather linear behavior during the initial stage of deterioration.

Therefore, the hyperbolic tangent was used as deterioration model in this case, too.

In Fig. 10, the squared residuals of the deterioration models for the LPC efficiency are shown. Since for prediction purposes the accuracy of the deterioration model during the advanced stage of deterioration is more important, the squared residuals are summed up backwards. The hyperbolic tangent yields a slightly smaller overall error compared to the power function. However, the power function performs better between 5 % to 100 % of the reference cycle number. Hence, for prediction this deterioration model is more adequate. The poor performance of the power function during the initial stage of deterioration can be explained by the deterioration behavior of the LPC efficiency, which is almost linear at the beginning. As can be seen in Fig. 4, this shape cannot be reproduced by the power function. Apart from that, based on the baseline deterioration trends it can be stated that deterioration is a continuous process. This becomes evident since none of the calculated baseline deterioration trends depicted in Fig. 9 levels out towards the end of the considered period of deterioration. Hence, the power function predicting unrestricted growth or decrease seems to be generally more adequate than the hyperbolic tangent, which is asymptotically tending towards a limit.

In Fig. 11 the power function fitted to the deterioration trend of the individual gas turbine "GT-A-02" is shown. Apparently, the HPC efficiency is subject to epistemic uncertainty which manifests as oscillation of the moving average around the deterioration model. In Fig. 12, the residual

$$(11) \quad \varepsilon_3(t, k) = \Delta\bar{m}(t, k) - \Delta\hat{m}(t - k), \quad t = k \dots n$$

is plotted over time. Apparently, the period of the oscillation matches one year. Hence, a possible root cause is the seasonal variation of the ambient temperature. Since the reduced high pressure spool speed is approximately constant due to the gas turbine control logic, this leads to changes of the corresponding mechanical spool speed, which in turn cause changes of the tip clearances within the high pressure system. Since the employed gas turbine model does not account for tip clearance changes, the calculated efficiencies exhibit seasonal variations, too: Due to the higher mechanical spool speeds and the smaller tip clearances, the HPC efficiency is higher during the hot summer months peaking at the beginning of August. The minimum of the seasonal variations occurs at the beginning of March. According to Fig. 11, despite the significant scatter and the considerable oscillation of the moving average the power function still seems to be a valid deterioration model in case of individual gas turbines, too. Defined by the NGV

throat area, the HPT capacity is not subject to tip clearance effects.

Prediction of Health Parameters

The prediction results, which were calculated based on the individual data set “GT-A-02”, are shown in Fig. 13. In this case the power function is very well suited for the prediction of the HPC efficiency: Extrapolating the raw data the remaining number of cycles can already be predicted with a maximum error of 3.1 % at 70 % of the reference cycle number. In comparison, given the configuration presented in [4] the method of Bayesian forecasting provides worse results, since it is less robust to the seasonal variations. Apparently, the incorporation of a priori information by usage of the power function increases the stability of the prognosis. In order to further increase the prediction accuracy, the systematic and seasonally occurring effects in the data set “GT-A-02” are eliminated. Therefore, the power function is fitted to the raw data. Then, the residuals are fitted using a sine function to determine the seasonal portion. Finally, the seasonal portion is subtracted from the raw data. Using this seasonally detrended data, prediction accuracy can be improved significantly as indicated by Fig. 13.

In order to increase the statistical relevance of the results, the described extrapolation method was applied to a variety of data sets. As can be seen in Fig. 14, for the most part the prognosis quality of the HPC efficiency is worse compared to the LPC efficiency. This can be attributed to the seasonal variations underlying the HPC efficiency, which cannot be observed in case of the LPC efficiency. Apparently, the prognosis quality could be significantly improved by inclusion of a tip clearance model in the performance analysis. The prognosis quality of the HPT is worst. This is mainly due to the low total number of operating cycles, which is only 40 % of the reference cycle number. As indicated by the rapid decrease of the prediction error of the efficiency between 20 % and 40 % of the reference cycle number, a certain amount of cycles is needed to get stable extrapolation results. Based on this results it can be stated that given an optimum fit the prediction gets more accurate the more severe the performance deterioration is and the smaller the epistemic uncertainties of the health parameters are. Furthermore, the larger the total cycle number, the earlier accurate predictions can be made relative to the total cycle number.

Summary

Based on steady-state in-service data of two different groups of gas turbines performance deterioration trends were calculated for several gas turbine compo-

nents. For the evaluated data, the power function has proven to be a suitable deterioration model in case of power plant group baselines as well as individual gas turbines. Furthermore, this model is appropriate for the prognosis of deterioration trends of individual gas turbines. However, in order to get accurate results epistemic uncertainties need to be minimized and a certain minimum number of cycles are needed.

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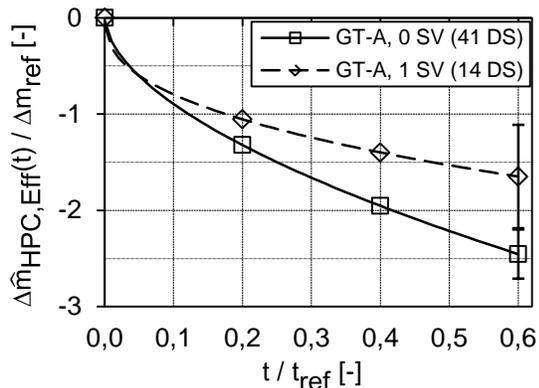


Fig. 1: Influence of the number of shop visits on the deterioration of the HPC efficiency

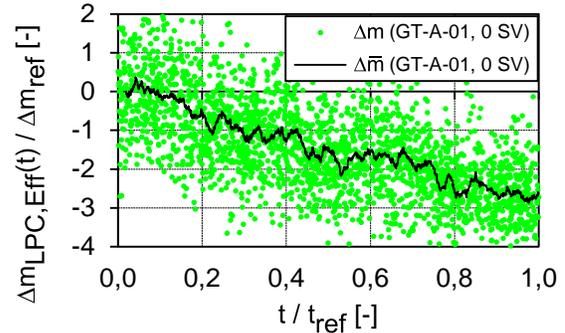


Fig. 2: Example plot of health parameter versus cycle number

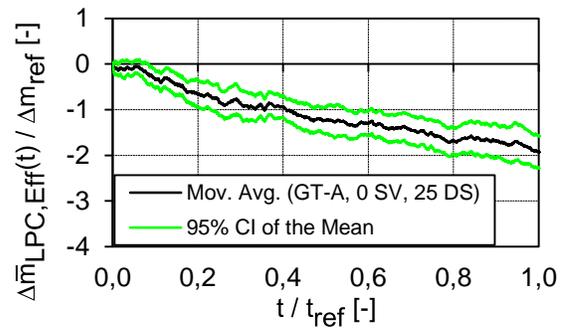


Fig. 3: Baseline deterioration of the LPC efficiency

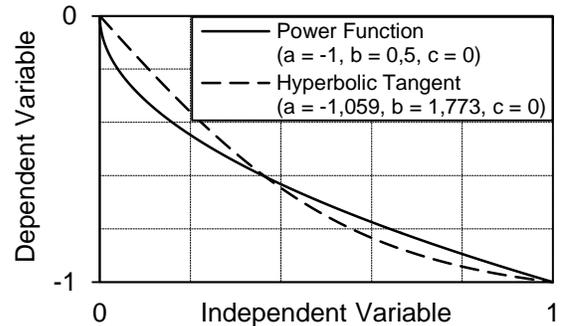


Fig. 4: Investigated statistical deterioration models

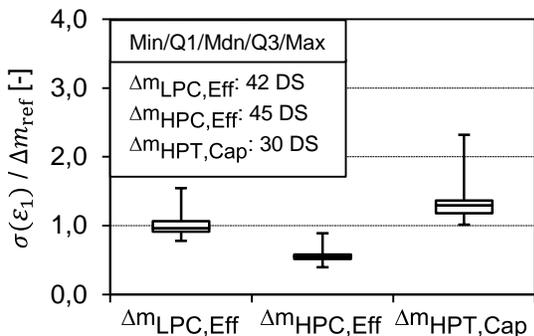


Fig. 5: Box plot of the estimated aleatoric uncertainty in health parameter calculation

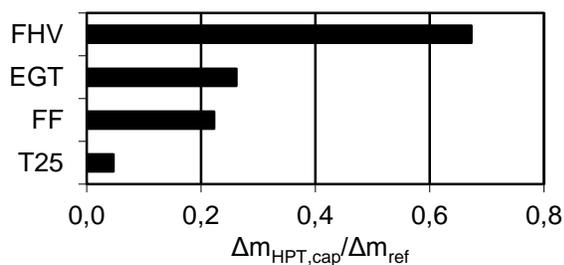


Fig. 6: Uncertainty in health parameter calculation due to measurement / modeling uncertainties

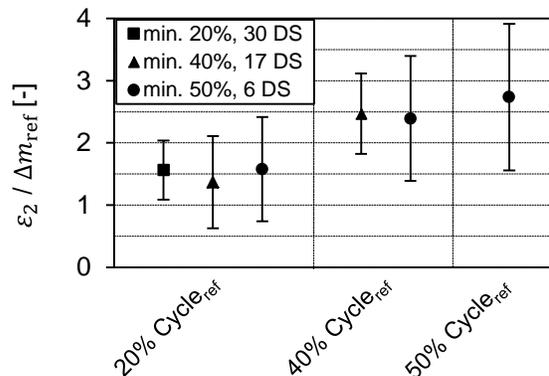
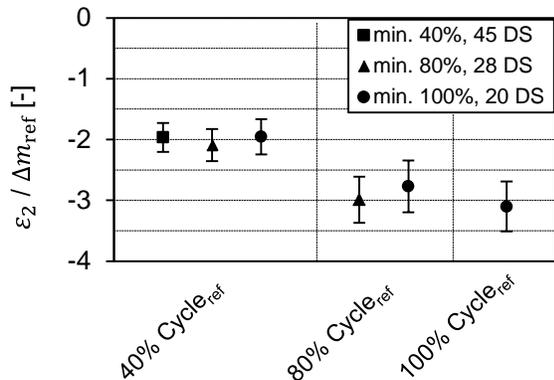
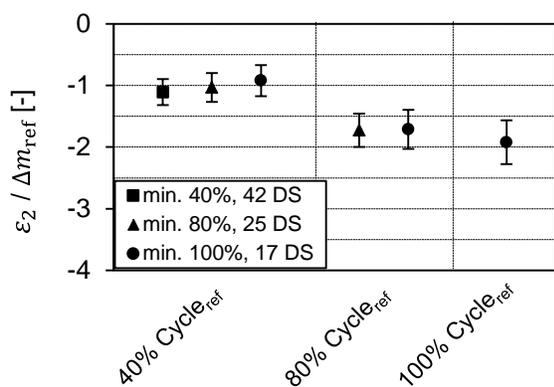
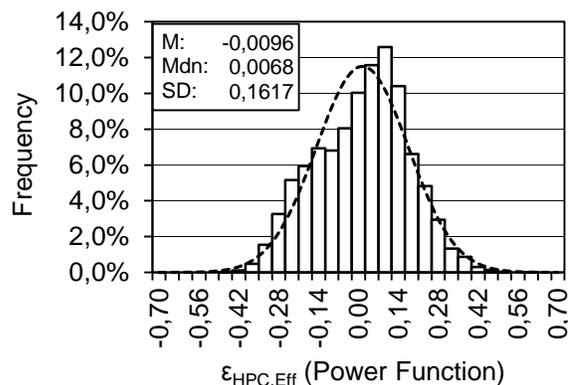
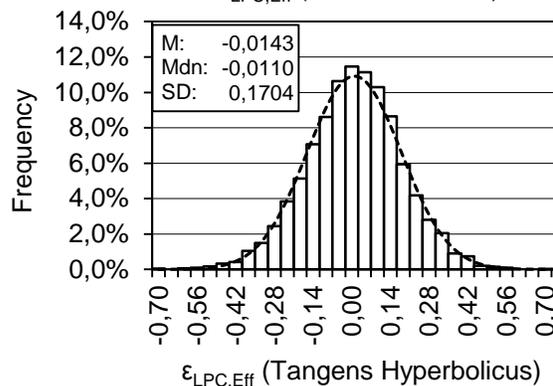
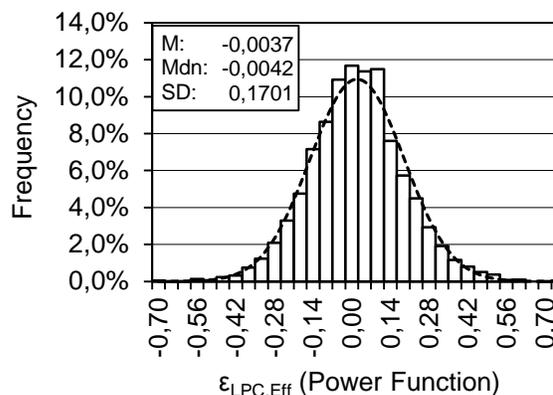


Fig. 7: Influence of the number of averaged data sets on the uncertainty of the baseline deterioration trends at selected instants of the reference cycle number



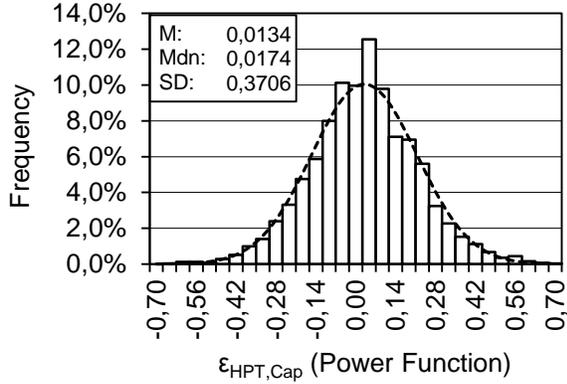


Fig. 8: Histograms of the residuals of the baseline deterioration trends

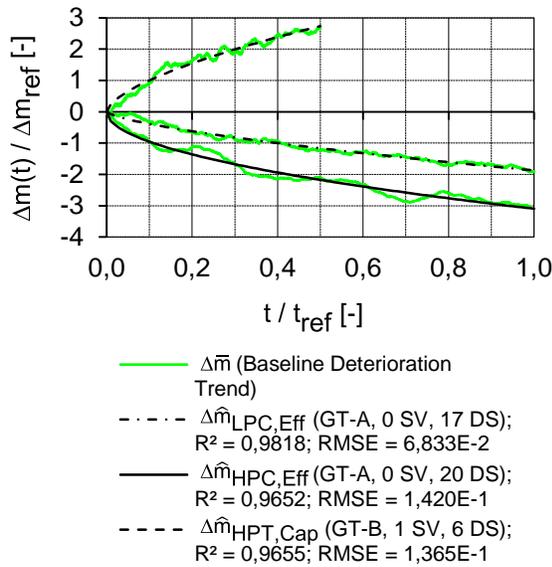


Fig. 9: Filtered baseline deterioration trends and the fitted power functions

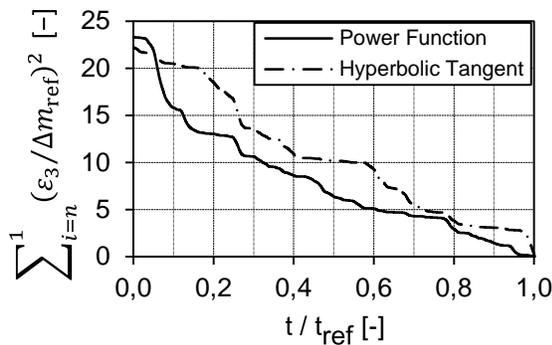


Fig. 10: Reversely summed up squared residuals versus number of cycles for the different deterioration models of the LPC efficiency

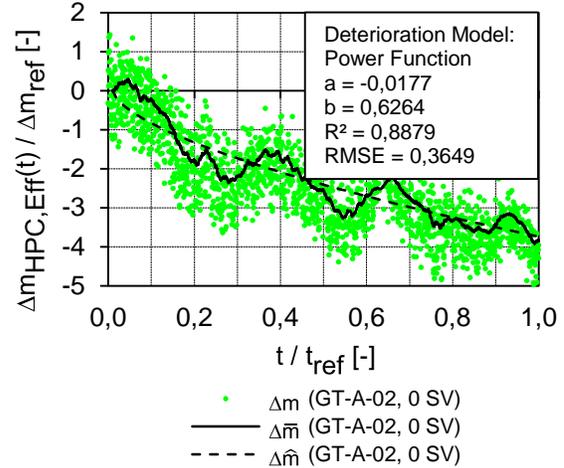


Fig. 11: Deterioration trend of the HPC efficiency

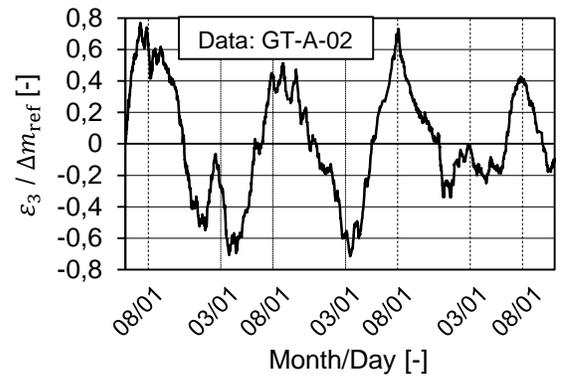


Fig. 12: Seasonal variation of the moving average of the HPC efficiency

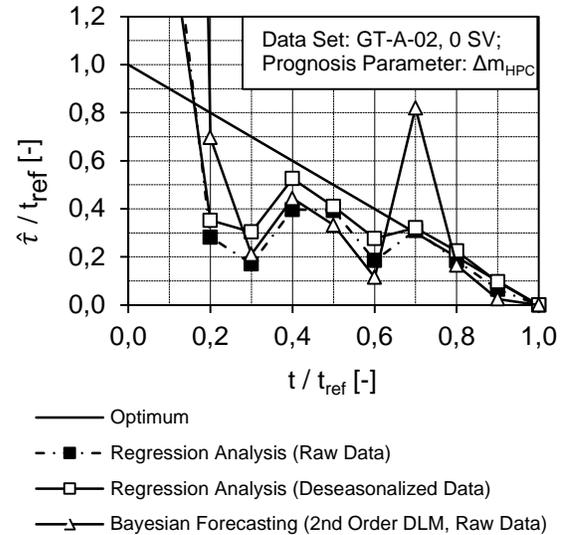


Fig. 13: Comparison of the predictive capabilities of the regression method and the Bayesian forecasting approach

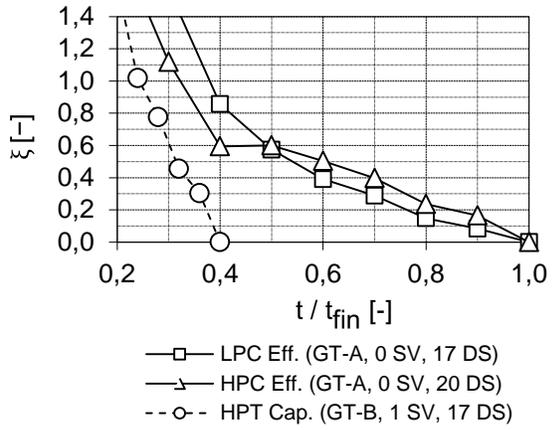


Fig. 14: One-sided 95 % confidence intervals of the relative prediction error of the final health parameter value at given instants of the total cycle number